Two Useful Right Triangle Theorems

These theorems usually (or *should!*) appear in most Geometry textbooks.
These are valuable mathematically and educationally. They connect geometry of similar triangles, proportional reasoning, algebra, and number relationships.

1. The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments into which the altitude separates the hypotenuse.
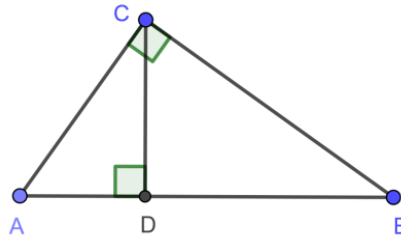
   Given:

   Right $\triangle ABC$ with right $\angle C$

   $\overline{CD} \perp \overline{AB}$

   Show:

   $$\frac{AD}{CD} = \frac{CD}{DB}$$

2. If the altitude to the hypotenuse is drawn in a right triangle, the length of either leg is the geometric mean of the length of the hypotenuse and the segment on the hypotenuse which is adjacent to that leg.

   Given:

   Right $\triangle ABC$ with right $\angle C$

   $\overline{CD} \perp \overline{AB}$

   Show:

   $$\frac{AB}{AC} = \frac{AC}{AD} \quad \text{and} \quad \frac{AB}{BC} = \frac{BC}{DB}$$